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of a Torpedo.

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Admiralty Research Laboratory

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# The Effect of Changes in the Stability Derivatives on the Dynamic Behaviour of a Torpedo

By

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Summary.—This report investigates the extent to which the dynamic behaviour of a torpedo is sensitive to changes in its stability derivatives. The main object in carrying out the investigation was to provide guidance on the accuracy of measurement of the stability derivatives that should be necessary for any given torpedo. The considerations of the report are, however, also pertinent to the problem of deciding the effectiveness of possible changes in the design of a torpedo, the dynamic behaviour of which is unsatisfactory. Illustrative examples are worked out in detail. The report emphasises the importance of the so-called margin of stability.

- 1. Introduction.—The purpose of this report is to investigate the extent to which the dynamic behaviour of a torpedo is sensitive to changes in its stability derivatives. Since dynamic behaviour covers the whole class of possible motions of a torpedo, attention has had to be confined to certain well defined aspects. The main object in carrying out the investigation was to provide guidance on the accuracy of measurement of the stability derivatives that would be necessary for any given torpedo: specifically, what error in predicted performance will given errors in the stability derivatives cause? The considerations of the report are, however, also pertinent to the problem of deciding the effectiveness of possible changes in the design of a torpedo, the dynamic behaviour of which is unsatisfactory.
- 2. The Motion of the Torpedo. We consider motion in a vertical plane only, and neglect buoyancy and trim effects. The treatment applies equally to motion in a horizontal plane only.

The relevant equations of motion are

$$Z_{s}z + Z_{s}b + Z_{s}\delta_{s} - m_{s}V\delta_{s} - m_{t}Vb, \qquad (1)$$

$$M_{s}z + M_{s}b + M_{s}\delta_{s} - J_{s}b, \qquad (2)$$
where
$$V \qquad \text{speed of torpedo, assumed constant}$$

$$z \qquad \text{angle of attack}$$

$$Accession For$$

θ pitch angle
 δ elevator angle
 q θ pitching rate
 Z denotes the coefficient of a force normal to the torpedo axis

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M denotes the coefficient of a moment about the transverse horizontal axis through the torpedo c.g.

 $Z_x = \frac{\partial Z}{\partial x}$ , etc.

 $m_1$  total transverse mass of torpedo  $\approx m + K_1 m_1$ 

 $m_1$  - total longitudinal mass of torpedo  $\simeq m + K_1 m_1$ 

 $m \rightarrow \text{mass of torpedo}$ 

 $m_t = \text{mass of displaced fluid}$ 

 $J_y =$ total moment of inertia about the transverse horizontal axis through the c.g.  $\approx I_y + K'I_{yt}$ 

 $I_v$  — moment of inertia of torpedo about the transverse horizontal axis through the c.g.

 $I_{yf}$  = moment of inertia of displaced fluid about the transverse horizontal axis through the c.g.

 $K', K_1, K_2$  are Lamb's inertia coefficients for an equivalent ellipsoid.

The positive senses of the various parameters are illustrated in Fig. 1.

If we multiply each term of equations (1) and (2) by  $e^{-rt}$  and integrate with respect to the time t between 0 and  $\infty$  throughout (denoting Laplace-transformed quantities by a bar) and eliminate  $\theta$  we have

$$[m_{2}VJ_{\gamma}p^{2} + (J_{\gamma}Z_{\alpha} + m_{2}VM_{\gamma})p + M_{\gamma}Z_{\gamma} + M_{z}(m_{1}V + Z_{\gamma})]\bar{z}$$

$$= [J_{\gamma}Z_{\alpha}p + M_{\alpha}(m_{1}V + Z_{\gamma}) \cdots M_{\gamma}Z_{\alpha}]\delta_{z}. \qquad (3)$$

If we had found, instead, the equation connecting  $\bar{z}_0$  or  $p\theta$  with  $\delta_n$ , the left-hand side would have been identical with that of equation (3). We write this left-hand side as

where

$$\begin{cases}
A_{1}p^{2} + A_{2}p + A_{3}]\bar{x}, \\
A_{1} = m_{2}VJ, \\
A_{2} = -J, Z_{x} - m_{2}VM_{y}, \\
A_{3} + M_{g}Z_{x} - M_{x}(m_{1}V + Z_{g})
\end{cases}$$
(4)

It follows from equation (3) that the transient part of the solution for  $\alpha(t)$  will be

where  $\mu_1$  and  $\mu_2$ , the decay constants of the motion are the roots of

and  $\lambda_1$  and  $\lambda_2$  are constants.

In particular, if the elevators are locked at zero, the right-hand side of equation (3) disappears, and the expression (5) represents the complete solution for the angle of attack  $\alpha$ , following a disturbance.

A torpedo is said to have dynamic stability, if, when disturbed from a straight-line path, it will again settle down to a straight-line path (but not necessarily the original straight-line path), that is, it tends to reduce its angle of attack to zero. If a dynamically unstable torpedo is disturbed from its straight-line path, it will circle with smaller and smaller radius until the linear analysis used here no longer applies. It is clear from equation (5) that the necessary and

sufficient condition for the torpedo to have dynamic stability is that the roots of equation (6) have negative real parts. The necessary and sufficient condition for this is that  $A_1$ ,  $A_2$  and  $A_3$  all have the same sign:

$$A_1 = m_2 V J_y > 0$$
  
 $A_2 = -J_y Z_x - m_2 V M_y > 0$ 

since  $Z_{\alpha} < 0$ ,  $M_{q} < 0$  for all conventional torpedoes. The criterion for dynamic stability is therefore that  $A_{3} > 0$ . Since  $Z_{\alpha}M_{q} > 0$ , we can write

$$G = 1 - \frac{M_s(m_1V + Z_g)}{Z_sM_g} > 0 \text{ for dynamic stability }. \qquad (7)$$

G is called the margin of stability. The following Table indicates torpedo behaviour for different values of G.

	į	$\boldsymbol{G}$	Controllability	Application
Dynamically unstable Marginally stable		<0	Requires special control equipment	No known application.
Dynamically stable		0·1 0·2 0·3 0·4	Turns rapidly with small rudders; bard to control and maintain in straight flight.	Homing torpedoes.
	:	0·5 0·6	Furns rapidly with medium-sized rudders; controls moderately well.	Homing torpedoes and straight-running torpedoes
		0·7 0·8	Turns rapidly with large rudders; controls casily.	Straight-running torpedoes.
	;	0·9 1·0 >1·0	Requires very large rudders: controls very easily.	Straight-running torpedoes.

2.1. Circling Motion.—Suppose the torpedo is moving steadily in a vertical circle of constant radius R, with the following (constant) values of its parameters

$$q = \theta = \theta^*; \quad \alpha = \alpha^*; \quad \delta_c = \delta_c^*$$
  
 $\dot{\alpha} = \dot{\theta} = 0.$ 

Putting these values in equations (1) and (2) and solving for  $\theta^*$  and  $x^*$  we have

$$G\frac{\theta^*}{\delta_{,*}} = \frac{M_a Z_{\delta_c} - Z_a M_{\delta_c}}{Z_a M_q} \qquad (8)$$

(We note that, since the right-hand side of equations (8) and (9) are both negative for all conventional torpedoes,

$$\operatorname{sgn}\frac{\theta^*}{\delta_{\cdot}^*} = \operatorname{sgn}\frac{\alpha^*}{\delta_{\cdot}^*} = -\operatorname{sgn}G.$$

This implies that a dynamically stable torpedo (G > 0) turns with its elevators, while a dynamically unstable one (G < 0) turns against its elevators.)

In a stable (G > 0) turn of constant radius R,  $V = R\theta^*$ , and from equation (8) we have

$$R = \frac{VGZ_{x}M_{q}}{M_{x}Z_{\delta_{c}} - Z_{a}M_{\delta_{c}}} \frac{1}{\delta_{a}^{*}} \dots \qquad (10)$$

- 3. The Effect of Errors in the Stability Derivatives.—We can now study the effects of errors in the stability derivatives  $Z_a$ ,  $M_a$ ,  $Z_a$ ,  $M_a$ ,  $Z_a$ , and  $M_a$  on three aspects of the dynamic behaviour of a torpedo:
  - (a) The effect on the radius of turn R for a given elevator angle  $\delta_{*}$
  - (b) The effect on the margin of stability G
  - (c) The effect on the transient motion of the torpedo following a disturbance. This is done by studying the effect on the decay constants  $\mu_1$  and  $\mu_2$  defined by equation (5).

Errors in the range  $\pm 20$  per cent will be considered for the static and control surface derivatives  $Z_*$ ,  $M_*$ ,  $Z_*$ , and  $M_*$ , and errors in the range  $\pm 50$  per cent for the rotary derivatives  $Z_*$  and  $M_*$ . Each case will be illustrated by examples of two torpedoes of widely differing hydrodynamic characteristics, Torpedo A (G about 1.0), and Torpedo B (G about 0.6). They have the following hydrodynamic coefficients:

#### TORPEDO A.

$$\frac{\partial C_L}{\partial \alpha} = -3.09; \qquad \frac{\partial C_L}{\partial \delta_i} = -0.70; \qquad \frac{\partial C_L}{\partial (l/R)} = -1.40s$$

$$\frac{\partial C_M}{\partial \alpha} = -0.05_{\delta}; \qquad \frac{\partial C_M}{\partial \delta_i} = -0.37; \qquad \frac{\partial C_M}{\partial (l/R)} = -0.63_{\delta}.$$

We use the relations

$$Z_{\alpha} = \frac{1}{2}\rho A V^{2} \frac{\partial C_{L}}{\partial \alpha}; \qquad Z_{\delta_{\alpha}} = \frac{1}{2}\rho A V^{2} \frac{\partial C_{L}}{\partial \delta_{\alpha}}; \qquad Z_{q} = \frac{1}{2}\rho A V l \frac{\partial C_{L}}{\partial (l/R)};$$

$$M_{\alpha} = \frac{1}{2}\rho A V^{2} l \frac{\partial C_{M}}{\partial \alpha}; \qquad M_{\delta_{\alpha}} = \frac{1}{2}\rho A V l \frac{\partial C_{M}}{\partial \delta_{\epsilon}}; \qquad M_{q} = \frac{1}{2}\rho A V l^{2} \frac{\partial C_{M}}{\partial (l/R)};$$

where

 $\rho$  = density of water = 2 slugs/cu ft

 $A = \text{maximum cross-sectional area of torpedo} = 2.4 \text{ ft}^{2}$ 

V = speed of torpedo = 40 ft/sec

l =length of torpedo = 14 ft.

This gives

$$\frac{Z_a}{10^3} = -11.866; \qquad \frac{Z_a}{10^3} = -2.688; \qquad \frac{Z_q}{10^3} = -1.888$$

$$\frac{M_a}{10^3} = -2.957; \qquad \frac{M_b}{10^3} = -19.891; \qquad \frac{M_q}{10^3} = -11.967.$$

Also

m = mass of torpedo = 58.5 slugs

 $I_s$  = moment of inertia of torpedo about the transverse horizontal axis through the c.g. = 745 slugs/ft<sup>3</sup>.

The Lamb inertia coefficients for an ellipsoid of the same fineness ratio (8) are

$$K_1 = 0.029$$
;  $K_2 = 0.945$ ;  $K' = 0.840$ , giving  $\frac{m_1 \Gamma}{10^3} = \{-2.408$ ;  $\frac{m_2 \Gamma}{10^3} = \{-4.551$ ;  $\frac{f_v}{10^3} = \{-1.371$ .

#### TORPEDO B.

These give

3.1. The Effect on Radius of Turn. – For a given elevator deflection  $\delta_i^*$ , the radius of turn is (equation (10)).

where 
$$R' = \frac{GZ_{\tau}M_{\tau}}{M_{\tau}Z_{\delta_{\tau}} - Z_{\tau}M_{\delta_{\tau}}} = \frac{Z_{\tau}M_{\tau}}{M_{\tau}Z_{\delta_{\tau}} - Z_{\tau}M_{\delta_{\tau}}} \left\{ \begin{array}{c} \vdots \\ M_{\tau}Z_{\delta_{\tau}} - Z_{\tau}M_{\delta_{\tau}} \end{array} \right\}. \qquad (11)$$

We denote by  $R_0$  and  $R_0'$  the values of R and R' when there are no errors in the stability derivatives, and by  $\delta R$  and  $\delta R'$  the changes in R and R' due to changes  $\delta C$  in C, where C is one of  $Z_2$ ,  $M_2$ ,  $Z_2$ ,  $M_3$ ,  $Z_4$  and  $M_4$ .

Since V and  $\delta_i^*$  are constant, it is clear that

$$\frac{\delta R}{R} = \frac{\delta R'}{R'}$$
.

The fractional change in R for any given fractional error in C can be calculated from equation (11) as set down below, for all six interpretations of C.

We note that  $R_0$  has the following values for the two torpedoes chosen as examples:

Torpedo A  $R_0 = 144$  ft when  $\delta_i^* = 10 \deg$ 

Torpedo B  $R_0 = 100$  ft when  $\delta_s^* = 10$  deg.

Errors in Z.

$$\frac{\delta R}{R} = \frac{M_*[M_*Z_{s_*} - M_{s_*}(m_1V + Z_{s})]}{M_*Z_{s_*} - M_{s_*}(m_1V + Z_{s})} \frac{\delta Z_{s}|Z_{s_*}}{M_*Z_{s_*} - M_{s_*}(1 + \frac{\delta Z_{s}}{Z_{s_*}})}$$
Torpedo A:
$$\frac{\delta R}{R} = \frac{\delta Z_{s}|Z_{s_*}}{-21 \cdot 95 - 22 \cdot 71} \frac{\delta Z_{s_*}}{Z_{s_*}}$$
Torpedo B:
$$\frac{\delta R}{R} = \frac{\delta Z_{s}|Z_{s_*}}{+0 \cdot 92 - 0 \cdot 64} \frac{\delta Z_{s_*}}{Z_{s_*}}$$
Errors in M.

$$\frac{\delta R}{R} = \frac{Z_{s}[M_*Z_{s_*} - M_{s_*}(m_1V + Z_{s})]}{M_*Z_{s_*} - M_{s_*}(m_1V + Z_{s})} \frac{\delta M_*|M_*}{Z_{s_*} - M_{s_*}(1 + \frac{\delta M_*}{M_*})}$$
Torpedo A:
$$\frac{\delta R}{R} = \frac{\delta M_*/M_*}{+21 \cdot 95 - 0 \cdot 77} \frac{\delta M_*}{M_*}$$
Torpedo B:
$$\frac{\delta R}{R} = \frac{\delta M_*/M_*}{-0 \cdot 92 - 0 \cdot 27} \frac{\delta M_*}{M_*}$$
Errors in Z,

$$\frac{\delta R}{R} = \frac{\delta Z_{s}/Z_{s_*}}{M_*Z_{s_*} - 1 - \frac{\delta Z_{s_*}}{Z_{s_*}}}$$
Torpedo A:
$$\frac{\delta R}{R} = \frac{\delta Z_{s}/Z_{s_*}}{-2 \cdot \sqrt{Z_{s_*}}}$$
Torpedo B:
$$\frac{\delta R}{R} = \frac{\delta Z_{s}/Z_{s_*}}{-2 \cdot \sqrt{Z_{s_*}}}$$
Torpedo A:
$$\frac{\delta R}{R} = \frac{\delta Z_{s}/Z_{s_*}}{-2 \cdot \sqrt{Z_{s_*}}}$$
Errors in M<sub>s</sub>,
$$\frac{\delta R}{R} = \frac{\delta M_*/M_{s_*}}{-3 \cdot 38 - \frac{\delta Z_{s_*}}{Z_{s_*}}}$$
Errors in M<sub>s</sub>,
$$\frac{\delta R}{R} = \frac{\delta M_*/M_{s_*}}{-2 \cdot \sqrt{M_{s_*}} - \frac{\delta M_{s_*}}{M_{s_*}}}$$
Errors in M<sub>s</sub>,
$$\frac{\delta R}{R} = \frac{\delta M_*/M_{s_*}}{-2 \cdot \sqrt{M_{s_*}} - \frac{\delta M_{s_*}}{M_{s_*}}}$$
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Errors in M<sub>s</sub>,
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 $\delta M_{\gamma_{i}}$ 

 $M_{s_2}$ 

1.42

R

Errors in 
$$Z_q$$

$$Z_i \rightarrow Z_s + \delta Z_s$$

$$\frac{\delta R}{R} = \frac{-M_{*}Z_{*}}{M_{*}Z_{*} - M_{*}(m_{1}V + Z_{q})} \frac{\delta Z_{*}}{Z_{*}} = \frac{Z_{q}}{(m_{1}V + Z_{q})} \frac{G_{0} - 1}{G_{0}} \frac{\delta Z_{q}}{Z_{q}},$$

where  $G_0$  is the value of G, the margin of stability, when there are no errors in the stability derivatives.

Torpedo A: 
$$\frac{\delta R}{R} = -0.04 \frac{\delta Z_q}{Z_s}$$

Torpedo B: 
$$\frac{\delta R}{R} = + 0.91 \frac{\delta Z_q}{Z_q}$$
.

## Errors in $M_q$

$$M_a \rightarrow M_s + \delta M_a$$

$$\frac{\delta R}{R} = \frac{M_{a}Z_{a}}{M_{q}Z_{a} - M_{a}(m_{1})} \cdot \frac{\delta M_{q}}{+ Z_{q}} \cdot \frac{\delta M_{q}}{M_{q}} = \frac{1}{G_{0}} \cdot \frac{\delta M_{q}}{M_{q}}$$

Torpedo A: 
$$\frac{\delta R}{R} = +0.99 \frac{\delta M_g}{M_s}$$

Torpedo B: 
$$\frac{\delta R}{R} = +1.80 \frac{\delta M_g}{M_g}$$
.

These results are plotted in the form percentage error in R against percentage error in C in Fig. 2 for Torpedo A, and in Fig. 3 for Torpedo B. It is clear from Fig. 2 that, for Torpedo A, errors only in  $M_q$  and  $M_{\delta_q}$  are significant. It is therefore useful to study the variation of R when there are errors in  $M_q$  and  $M_{\delta_q}$  simultaneously. The result for Torpedo A is

$$\frac{\delta R}{R} = \frac{\left[\begin{array}{cc} 0.99 \frac{\delta M_q}{M_s} & 1.04 \frac{\delta M_s}{M_{s_s}} \\ 1 + 1.04 \frac{\delta M_s}{M_{s_s}} \end{array}\right]}{1 + 1.04 \frac{\delta M_s}{M_{s_s}}}$$

This can be plotted as a family of straight lines in the  $\delta R/R = \delta M_s/M_s$  plane with  $\delta M_s/M_{s_s}$  as parameter. From this it can be seen what ranges of errors (positive and negative) in  $M_{s_s}$  and  $M_s$  are permissible for a given permissible range of error in R. This information is plotted in Fig. 4.

For Torpedo B errors in all stability derivatives are significant, and there is no point in considering simultaneous variations of two only.

3.2. The Effect on the Margin of Stability .-- G was defined by equation (7) as

$$G_0 = 1 - \frac{M_a(m_1V + Z_q)}{Z_aM_q}$$
,

where  $G_0$  is the value of G when there are no errors in the stability derivatives. We are now interested in the value of G when errors in the stability derivatives exist, and not in the fractional change in G. The values of  $G_0$  for the two torpedoes being considered are

Torpedo A: 
$$G_0 = +1.011$$

Torpedo B: 
$$G_0 = +0.556$$
.

Errors in 
$$Z_{\alpha}$$

$$Z_{\alpha} \rightarrow Z_{\alpha} + \delta Z_{\alpha}$$

$$G = 1 + \frac{G_0 - 1}{1 + \frac{\delta Z_*}{Z_*}}$$

Torpedo A: 
$$G = 1 +$$

$$G = 1 + \frac{0.011}{1 + \frac{\delta Z_a}{Z_a}}$$

$$G = 1 - \frac{0.444}{1 + \frac{\delta Z_*}{Z_*}}.$$

## Errors in $M_{\alpha}$

$$M_{\alpha} \rightarrow M_{\alpha} + \delta M_{\alpha}$$

$$G = G_0 + (G_0 - 1) \frac{\delta M_x}{M_x}$$

Torpedo A: 
$$G = 1.011 + 0.011 \frac{\delta M_a}{M_a}$$

Torpedo B: 
$$G = 0.556 - 0.444 \frac{\delta M_{\pi}}{M_{\pi}}$$
.

## Errors in $Z_q$

$$Z_i \rightarrow Z_q + \delta Z_q$$

$$G = G_0 + \frac{Z_q}{m_1 V + Z_q} (G_0 - 1) \frac{\delta Z_q}{Z_q}$$

Torpedo A: 
$$G = 1.011 - 0.040 \frac{\delta Z_q}{Z_q}$$

Torpedo B: 
$$G = 0.556 + 0.050 \frac{\delta Z_q}{Z_q}$$
.

## Errors in $M_a$

$$M_q \rightarrow M_q + \delta M_q$$

$$G = 1 + \frac{G_0 - 1}{1 + \frac{\partial M_g}{M_g}}$$

Torpedo A : 
$$G = 1 + \frac{0.011}{1 + \frac{\delta \dot{M}_g}{M_s}}$$

Torpedo B: 
$$G=1-\frac{0\cdot 444}{1+\frac{\delta M_q}{M_q}}$$
, which is the same variation as for  $\frac{\delta Z_z}{Z_z}$ .

These results are plotted with  $\delta C/C$  as a percentage in Fig. 5 for Torpedo A and in Fig. 6 for Torpedo B. It is obvious from the form of the equations that the variation of G with errors in the derivatives decreases as  $G_0$  approaches unity and is in fact zero at  $G_0 = 1$ .

3.3. The Effect on the Transient Motion of the Torpedo, Following a Disturbance.—It was shown in Section 2 that the transient part of the solution for the angle of attack  $\alpha(t)$  following a disturbance was the expression (5):

$$\lambda_1 e^{\mu_1 t} + \lambda_2 e^{\mu_2 t}$$
.

The transient solution for the depth  $z_0$ , or pitching rate  $\theta$  would be of the same form, with of course, different values of the constants  $\lambda_1$  and  $\lambda_2$ . Real values of  $\mu_1$  and  $\mu_2$  will be associated with aperiodic motion, and imaginary values with oscillatory motion.

The effect of errors in the stability derivatives on the transient motion of the torpedo can be studied in two sub-sections :

- (a) The effect of such errors on the decay constants  $\mu_1$  and  $\mu_2$
- (b) The effect of such errors on the transient motion following one particular disturbance which will be taken as a step function input to the elevators.

3.3 (a).—The effect of errors on the decay constants.—The decay constants were defined by equations (4) and (6). It is obvious from these that there are two types of problem involved since errors in  $Z_q$  or  $M_z$  cause  $A_z$  only to vary, while errors in  $Z_a$  or  $M_q$  cause both  $A_z$  and  $A_z$  to vary.

## Errors in $M_{\star}$

$$M_z \to M_z + \delta M_z$$

Let  $\mu$  be a root of the new equation (replacing equation (6))

$$A_1\mu^2 + A_2\mu + A_3 - (m_1V + Z_d)M_a\frac{\delta M_a}{M_a} = 0$$
.

Put  $\mu = y$  and  $\delta M_z/M_z = x$ , and this becomes the equation of a conin in the x-y plane. In conventional conic notation, it becomes

where

$$b_1 y^2 + 2g_1 x + 2f_1 y + c_1 = 0,$$

$$b_1 = + A_1 = m_2 V J_y$$

$$2g_1 = -M_a (m_1 V + Z_q)$$

$$2f_1 = + A_2 = -m_2 V M_q - J_z Z_z$$

$$c_1 = + A_3 - M_q Z_q - M_z (m_1 V + Z_q).$$

The discriminant A is, in conic notation,  $h_1^2 - a_1 b_1 = 0$ . Hence the equation above represents a parabola, providing the conic is non-degenerate (the case where the conic is degenerate is discussed below). The parabola passes through the points  $(0, \mu_1)$  and  $(0, \mu_2)$  and its axis is parallel to the x axis. Its vertex has an x co-ordinate of

$$\frac{f_1^2 - b_1 c_1}{2b_1 g_1} = -1 - \frac{(m_2 V M_q - J_y Z_z)^2}{4m_2 V J_y M_z (m_1 V + Z_z)}.$$

The value of the decay constants for any given error in  $M_x$ , say  $\delta M_x^*$ , are the values of y at which the line  $x = \delta M_x^*/M_x$  meets the parabola.

The parabola cuts the x axis at the point  $x = G_0/(1 - G_0)$ , y = 0, where  $G_0$  is the margin of stability calculated when no errors exist in any derivative. With this value of x, the torpedo is marginally dynamically stable. Moreover, the nearer  $G_0$  is to unity the smaller is the change in the decay constants for any given error. At  $G_0 = 1$ , the coefficient  $g_1$  in the equation of the parabola disappears, and this is the condition for the parabola to degenerate into a parallel line-pair in the direction of the x axis, which implies no change at all in the decay constants for errors in  $M_2$ . We assume that when no errors exist, the torpedo is dynamically stable, that is  $G_0 > 0$  and  $\mu_1$  and  $\mu_2$  negative. It follows that the parabola faces right or left according as  $G_0 \ge 1$ .

The parabola is plotted in Fig. 7 for Torpedo A, and in Fig. 8 for Torpedo B. It should be noticed that the horizontal scales of these diagrams are in units of  $\delta M_z/M_z$  and not  $(\delta M_z/M_z)$  per cent as in previous diagrams. The variations of the decay constants are greater for

Torpedo B than for Torpedo A, as is to be expected, since  $G_0$  is nearer unity for Torpedo A. In fact, for Torpedo A, over the range  $\frac{|\delta M_{\pi}|}{|M_{\pi}|} \le 0.2$  (i.e.,  $\pm 20$  per cent error), there is no noticeable change in the decay constants. For Torpedo B the change in the decay constants for the same range of  $\delta M_{\pi}/M_{\pi}$  is noticeable but not significant.

The torpedo is dynamically stable or unstable according as  $\mu_1$  and  $\mu_2$  have negative or positive real parts. When  $\mu_1$  and  $\mu_2$  become imaginary (i.e., in the region of the diagram past the vertex of the parabola), the motion hitherto aperiodic becomes oscillatory. That the oscillatory motion is, in fact, stable can be easily checked.

## Errors in $Z_a$

$$Z_q \rightarrow Z_q + \delta Z_q$$

Let  $\mu$  be a root of the new equation

$$A_1^{\prime}\mu^2 + A_2\mu + A_3 - M_2 Z_q \frac{\delta Z_q}{Z_q} = 0.$$

Put  $\mu = y$  and  $\delta Z_i/Z_i = x$  and we can write this in conic notation as before

$$b_2y^2+2g_2x+2f_2y+c_2=0$$
 ,

where

$$b_{2} = + A_{1} = m_{2}VJ_{y}$$

$$2g_{2} = -M_{\alpha}Z_{y}$$

$$2f_{2} = -A_{2} - -m_{2}VM_{q} - J_{y}Z_{\alpha}$$

$$c_{2} = + A_{3} - M_{y}Z_{\alpha} - M_{z}(m_{1}V + Z_{y}).$$

This is, again, a parabola passing through  $(0, \mu_1)$  and  $(0, \mu_2)$ . The x co-ordinate of the vertex is now

$$\frac{f_2^2 - b_2 c_2}{2b_2 g_2} = \left[ -1 - \frac{(m_2 V M_y - J_y Z_x)^2}{4m_2 V J_y M_x (m_1 V + Z_y)} \right] \frac{m_1 V + Z_y}{Z_y}.$$

It will meet the x axis where

$$x = \frac{G_0}{1 - G_0} \frac{m_1 V + Z_1}{Z_4}.$$

It is in fact the same parabola as before, but with the horizontal scale multiplied by a factor  $(m_1V + Z_q)_i Z_q$ . Minimum variation again occurs when  $G_0 = 1$ , when the parabola degenerates as before. The parabola is plotted in Fig. 7 for Torpedo A and Fig. 8 for Torpedo B. In both cases the variation of the decay constants is a little greater than for the  $M_z$  case but it is still negligible for Torpedo A and not very significant for Torpedo B in the range  $\binom{\delta Z_q}{Z_{-1}} \leq 0.2$ .

# Errors in M.

$$M_q \rightarrow M_q + \delta M_q$$

Let  $\mu$  be a root of the new equation

$$A_1\mu^2+\left(A_2-m_2VM_q\frac{\delta M_q}{M_d}\right)\mu+A_3+Z_zM_z\frac{\delta M_z}{M_d}=0$$
 .

Put  $y = \mu$  and  $x = \delta M_q/M_q$ . In conic notation the equation becomes

$$2h_3xy = \frac{1}{3}y^2 + 2g_3x + 2f_3y + c_3 = 0, \qquad (12)$$

where

The discriminant  $A = h_3^2 - a_3 b_3 = h_3^2 > 0$ , so the equation represents a conic which, if non-degenerate, is a hyperbola. (The case when the conic is degenerate will be discussed below). The equation of the asymptotes is got from this equation by adding a constant  $\kappa$  such that

Solving for x we get

$$c_a + \kappa := 2g_a \left( \frac{4f_a h_a - 2b_a g_a}{4h_a^2} \right) = 2g_a$$

since

$$4f_3h_3 - 2b_3g_3 - 4h_2^2$$
 from (13).

The asymptote pair has, therefore, the equation

$$2h_2xy + b_2y^2 + 2g_2x + 2f_4y + 2g_4 - 0 ... ... ... (14)$$

The absence of a term in  $x^2$  shows that one of the asymptotes is parallel to the x axis. The slope of the other one is therefore the tangent of the angle between them and is

$$\pm \frac{2\sqrt{(h_3^2-a_2b_3)}}{a_3+b_3} = \pm \frac{M_{_Y}}{J_{_Y}}.$$

From (14) we see that the point (--1, 0) lies on the asymptote pair, and since the horizonial asymptote is certainly not y = 0, the point (--1, 0) necessarily lies on the sloping asymptote, whose equation is therefore

$$y = \pm \frac{M_{\rm v}}{I_{\rm v}} (1 \pm x)$$
.

Since the hyperbola passes through the points  $(0, \mu_1)$  and  $(0, \mu_2)$  where  $\mu_1$  and  $\mu_2$  are negative, this asymptote must have a negative gradient, whence its equation is

$$y \rightarrow -\frac{M_y}{L}(1+x)$$
,

 $M_d$  being negative for all conventional torpedoes. The equation of the other asymptote is found by differentiating equation (14) and finding the value of y for which dy/dx vanishes. It is

$$y = -\frac{g_n}{h_n} = \frac{Z_n}{m_n} \cdot .$$

The horizontal asymptote has therefore the equation

$$y = \frac{Z_*}{m_* \Gamma}.$$

We note that the asymptotes intersect at (x\*, v\*), where

$$x^{\bullet} = \frac{\int_{\gamma} Z_{\pi} - m_{\pi} V M_{\pi}}{m_{\pi} V M_{\pi}}.$$

We can now draw the asymptotes directly, and we know, moreover, two points on the hyperbola, namely,  $(0, \mu_1)$  and  $(0, \mu_2)$ . There is one other point of interest on the hyperbola. From equation (12) the x axis cuts the hyperbola where

$$x = \frac{-c_0}{2g_0} = -G_0.$$

There are four possible configurations of the hyperbola depending on whether  $A^{\bullet}$ ; thand  $G_{\bullet} \gtrsim 1$ . These are shown in Fig. 9. If we use the fact that the intercepts on any straight line cut off between a hyperbola and its asymptotes are equal, it is possible to sketch in the hyperbola

with reasonable accuracy from a knowledge of its asymptotes, the points  $(0, \mu_1)$ ,  $(0, \mu_2)$  and  $(-G_n, 0)$ , which are known to be on it. In the case  $G_n = 1$ , as  $G_n = 1$ , the rate of variation of one decay constant decreases, while that of the other increases to the slope of the sloping asymptote. In the case  $G_n = 1$  it is clear that the variation of both decay constants decreases as  $G_n$  approaches unity. If  $G_n = 1$ , the hyperbola degenerates into its asymptotes, and only one decay constant varies.

The hyperbola for Torpedo A is shown in Fig. 10, and for Torpedo B in Fig. 11, and the stablety regions are shown for each. It is easily proved that the region of oscillatory motion is a region of stable motion. It is interesting to note that when  $G_n = 1$  it is impossible to reach a condition of oscillatory motion of the body by altering  $M_n$  only.

It is clear from these Figures that errors in  $M_s$  are far more significant as regards the decay companies, than are errors in  $Z_s$  and  $M_s$ . In fact an error of -60 per cent in  $M_q$  would cause Torpedo A to conflicte, and Torpedo B to become dynamically unstable.

# Knor in Z.

Alm te

Let p be a root of the new equation.

$$A_{int} = \{A_i \mid J(Z, \frac{\delta Z_i}{Z_i})_{ii} = A_i \in M(Z, \frac{\delta Z_i}{Z_i} = 0).$$

Putting  $\mu = \chi$  and  $\delta Z_i/Z_i = \mathbf{a}_i$  this equation becomes, in conic notation.

The 1- again a hyperbole and, in the same way as before, we find that the asymptotes have the equations

$$s = \frac{M_s}{J}$$
 (horizontal asymptote)  
 $s = \frac{Z_s}{m_s J}$  (1 -  $v_s$  (stoping asymptote).

They intersect in the point of all where

$$s^{\bullet} = \frac{m_i V M_{i,j}}{\int Z_i} \frac{f_i Z_i}{i}$$

where the x-axis cuts the hyperbola at  $\chi = -e/2g_{\psi} = G_{0}$ , as before, the remarks made about the significance of toxing a x-due of  $G_{0}$  close to unity still apply. The four configurations shown in the Arabic apply, if the new expression for  $\chi^{2}$  is used. The e-hyperbolae are plotted in Fig. 10 for terpode A and in the Highest for Torpode 15. The variations in the decay constants are still large that not so a roady as they were for errors in M, perticularly as regardedness using accuracy, since accuracy to no essuring  $M_{\phi}$ . For terpode A an error of -460 per cent would be necessary to cause metabolity and an error of -240 per cent to cause oscillatory motion. For Torpodo B, instability would occur when  $Z_{\phi}$ 

had an error of -60 per cent. For both torpedoes,  $x^*$  is positive for the  $Z_x$  case and negative for the  $M_x$  case. This implies that the sloping asymptote has a less steep gradient in the  $Z_x$  case than in the  $M_x$  case, and that the variations in the decay constants are correspondingly less.

3.3. (b). The effect of errors on the transient motion for one particular disturbance.—The disturbance will be taken as a step function input on the elevators. The subsequent solution for the angle of attack will be studied. The relevant equation is equation (3), where  $\delta_i(t)$  is now a step function of magnitude  $\delta_i^*$ . Then,

$$\delta_c(p) = \frac{1}{p} \delta_c^*,$$

and equation (3) gives,

$$\frac{\bar{\alpha}(p)}{\delta_{i}^{*}} = \frac{J_{i}Z_{s,p} + M_{s,i}(m_{1}V + Z_{g}) - M_{g}Z_{s,p}}{m_{2}VJ_{s,p}(p - \mu_{1})(p - \mu_{2})}$$

by the definition of  $\mu_1$  and  $\mu_2$ . Splitting the right-hand side into partial fractions we have

$$\frac{\bar{\alpha}(p)}{\delta_c^*} = \frac{\lambda_2}{p} + \frac{\lambda_1}{p - \mu_1} + \frac{\lambda_2}{p - \mu_2}, \quad \dots \qquad \dots \qquad \dots \qquad \dots$$
 (15)

where

where

$$\lambda_{3} := \frac{M_{\delta_{1}}(m_{1}V + Z_{q}) - M_{q}Z_{\delta_{q}}}{m_{2}VJ_{5}\mu_{1}\mu_{2}}$$

$$\lambda_{1} := \frac{J_{5}Z_{\delta_{1}}\mu_{1} + M_{\delta_{1}}(m_{1}V + Z_{q}) - M_{q}Z_{\delta_{q}}}{m_{2}VJ_{5}\mu_{1}(\mu_{1} - \mu_{2})}$$

$$\lambda_{2} := \frac{J_{5}Z_{\delta_{1}}\mu_{2} + M_{\delta_{1}}(m_{1}V + Z_{q}) - M_{q}Z_{\delta_{q}}}{m_{2}VJ_{5}\mu_{2}(\mu_{2} - \mu_{1})}$$
(16)

Inverse Laplace-transferming equation (15) gives

$$\frac{\alpha(t)}{\delta_t^{**}} = \lambda_3 + \lambda_1 e^{\mu_1 t} + \lambda_2 e^{\mu_3 t}.$$

Since we are interested only in the transient solution, and not in the steady-state solution (which is  $\lambda_k$ ), we divide by  $\lambda_k$  to get finally,

$$\frac{\alpha(t)}{\lambda_{2}\delta_{z}^{**}} = 1 + \lambda_{1}^{1} e^{n_{1}t} + \lambda_{2}^{1} e^{n_{2}t}$$

$$\lambda_{1}^{1} = \frac{\lambda_{1}}{\lambda_{3}} = \left[ \frac{J_{1}Z_{N}\mu_{1}}{M_{N_{1}}(m_{1}V_{1} + Z_{q})} - M_{q}Z_{N_{1}} + 1 \right] \frac{\mu_{2}}{\mu_{1} - \mu_{2}} + \dots$$

$$\lambda_{2}^{1} = \frac{\lambda_{2}}{\lambda_{3}} = \left[ \frac{J_{1}Z_{N}\mu_{2}}{M_{N_{1}}(m_{1}V_{1} + Z_{q})} - M_{q}Z_{N_{2}} + 1 \right] \frac{\mu_{1}}{\mu_{2} - \mu_{1}}$$
(17)

 $\mu_1$  and  $\mu_2$  are affected by errors in  $Z_2$ ,  $M_2$ ,  $Z_3$  and  $M_4$  as already shown.  $Z_4^1$  and  $Z_2^1$  are affected by v that in all six derivatives. It is therefore possible to study how the solution (16) varies with errors in each of the six stability derivatives, one at a time. This has been done for three

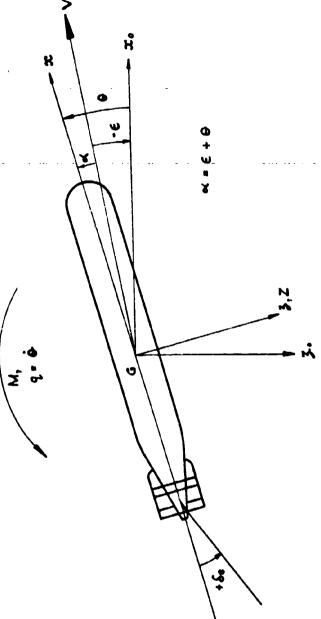
values of error in each derivative, namely  $0, \pm 50$  per cent for the rotary derivatives  $Z_q$  and  $M_q$ , and  $0, \pm 20$  per cent for the others. The results for Torpedo A are contained in Fig. 12, and for Torpedo B in Fig. 13. The time for the ordinate to reach 95 per cent of its final value is marked in each case. Errors in  $Z_{\delta_q}$  and  $M_{\delta_q}$  do not affect either torpedo noticeably. For the remaining derivatives, errors appear to affect Torpedo B more adversely than they do Torpedo A particularly in the case of the rotary derivatives  $Z_q$  and  $M_q$ . An error of -50 per cent in  $M_q$  causes a substantial change in the motion of Torpedo B. It should be noticed that the time is reach 95 per cent of the final value is less for Torpedo A than for Torpedo B; this is to be expected since Torpedo A has a larger margin of stability.

4. Summary and Conclusions.—In this report, the extent to which the dynamic behaviour of the torpedo is sensitive to changes in its stability derivatives has been investigated. Attention has necessarily been confined to certain well defined aspects of dynamic behaviour. These aspects were the radius of turn for a given elevator angle, the margin of stability, the decay constants of disturbed motion, and the motion following a particular disturbance, namely, a step function input to the elevators. It is not too unreasonable to suppose that these aspects are broadly representative of dynamic behaviour. It must be admitted, however, that the theoretical results apply to an uncontrolled torpedo. Nevertheless, it should be noted that according to the Table, the margin of stability indicates the ease with which a control system for a homing torpedo can be designed.

The results obtained in particular cases, namely, Torpedo A and Torpedo B which have been used as illustrative examples, may be summarised as follows: The radius of turn per elevator angle of Torpedo A is very susceptible to errors in  $M_{\delta_0}$  and  $M_{g}$ ; that of Torpedo B is very susceptible to errors in all derivatives except perhaps  $Z_{\delta_0}$ . The margin of stability G, for Torpedo A varies very little with errors in the stability derivatives. For Torpedo B, G varies rapidly with errors in  $Z_{\tau}$ ,  $M_{g}$  and  $M_{\chi}$ . For both torpedoes, the decay constants vary much more with errors in  $Z_{\tau}$  and  $M_{g}$  than with errors in  $M_{\tau}$  and  $Z_{g}$ . This tendency is reflected in the effect of errors on the solution for angle of attack following a step function input to the elevators, but it is not as pronounced as one would expect, presumably due to the effects of the errors on the coefficients  $\lambda_1^2$  and  $\lambda_2^2$ . For Torpedo A, the variation of the solution is small for all feasible errors. This is not so for Torpedo B, the variations due to errors in  $Z_{\tau}$  and  $M_{g}$  being rather severe.

In view of the complexity of the concept of dynamic behaviour and the number of parameters involved, it is difficult to draw general conclusions. It does seem clear, however, that the susceptibility of torpedo performance to changes or errors in the stability derivatives depends to a great extent on the margin of stability. The effect of errors is, in most respects, at a minimum when  $G_0 = 1$ , that is, when the torpedo is marginally statically stable.

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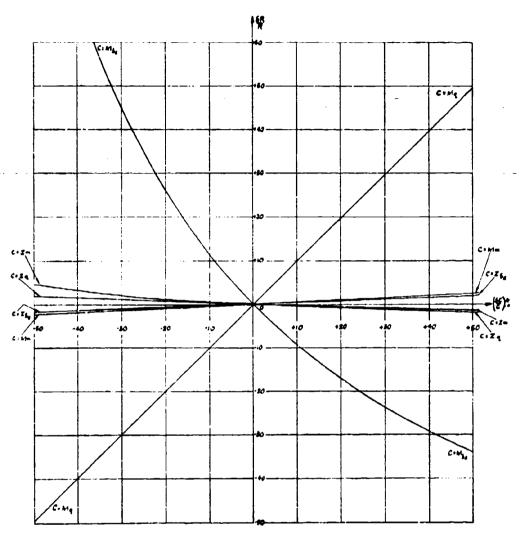


Fig. 2. Percentage error in radius of turn R against percentage error in hydrodynamic coefficients C (Forpedo A).

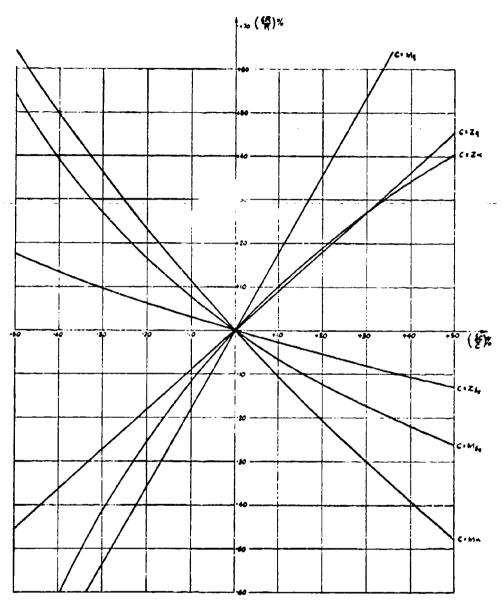


Fig. 3. Percentage error in radius of turn R against percentage error in hydrodynamic coefficients C (Torpedo A).

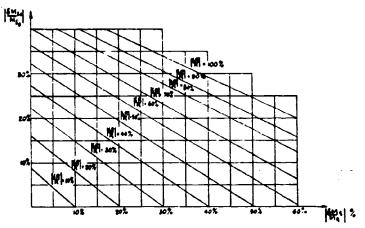


Fig. 4. Ranges for errors in  $M_{\theta_{\ell}}$  and  $M_{\theta}$  for various permissible errors in R (Torpedo A).

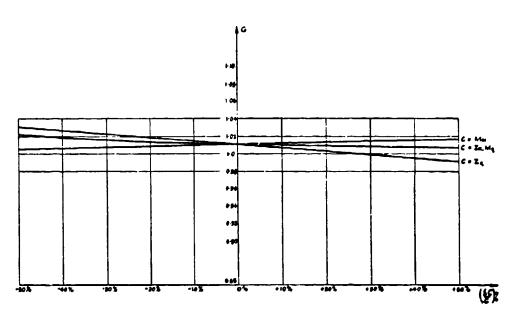


Fig. 5. The variation of G, the margin of stability with errors in the stability coefficients (Torpedo A).

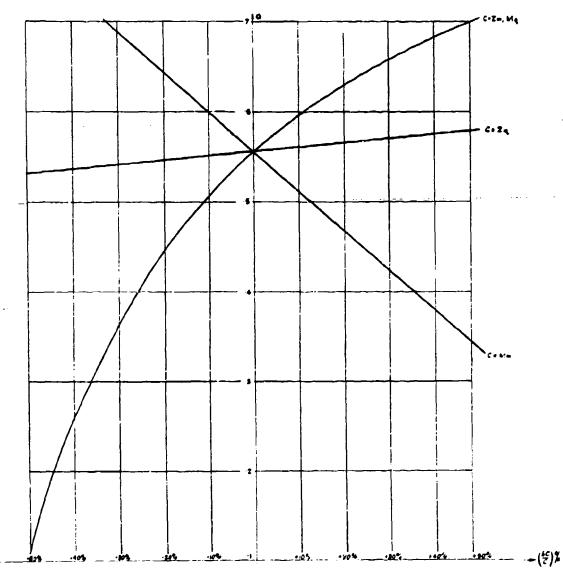


Fig. 6. The variation of G, the margin of stability, with errors in the stability derivatives (Torpedo  $\dot{\mathbf{B}}$ ).

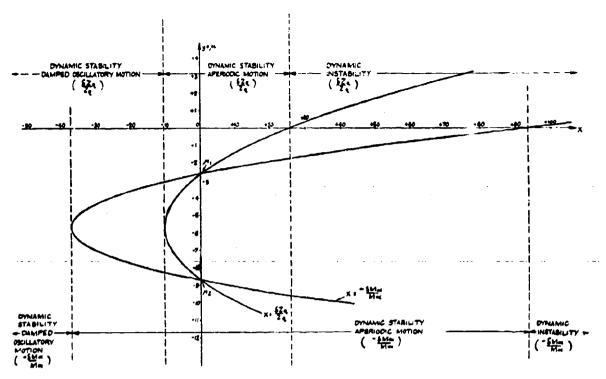


Fig. 7.—The variation of the decay constants  $\mu_1,\,\mu_2,\,$  with  $\delta Z_d,Z_d,\,$   $\delta M_{\pi},M_{\pi}$  (Torpedo A).

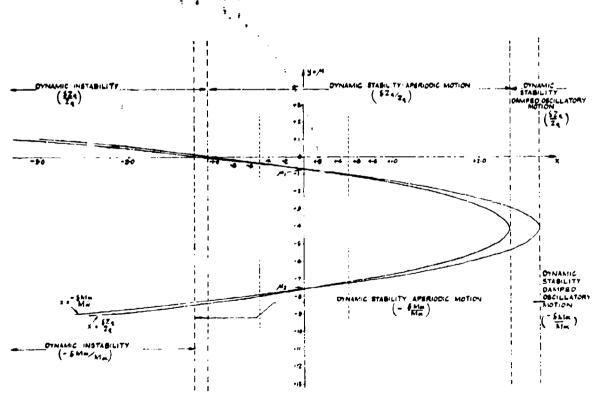
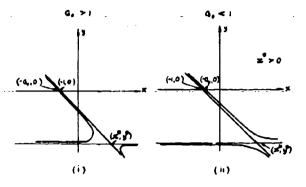


Fig. 8.—The variation of the decay constants  $\mu_1,\,\mu_2,\,$  with errors in  $Z_J$  and  $M_{\pi}$  (Torpedo B)



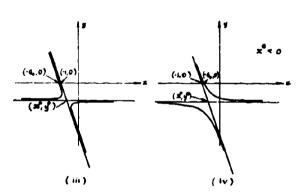


Fig. 9.

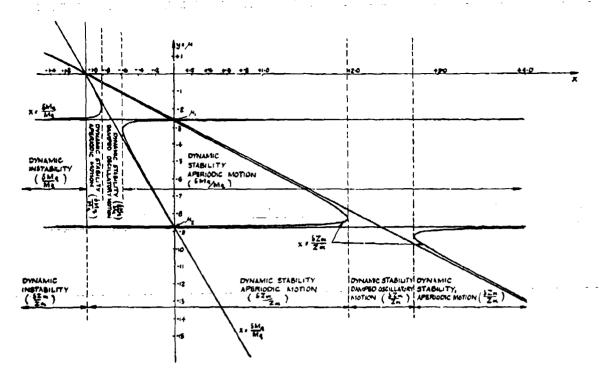


Fig. 10. Variation of decay constants  $\mu_1$  and  $\mu_2$  with  $\delta Z_{\mathbf{x}} Z_{\mathbf{z}}$  and  $\delta M_{\phi} M_{\phi}$  (Torpedo A).

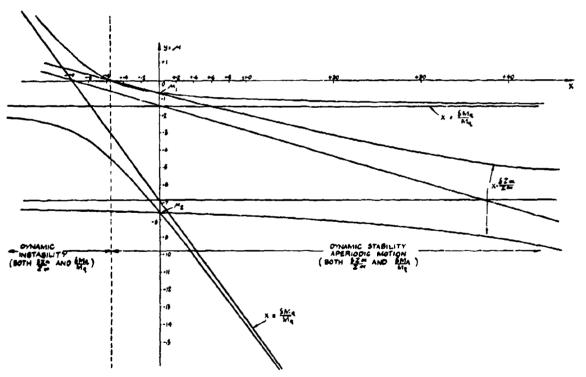
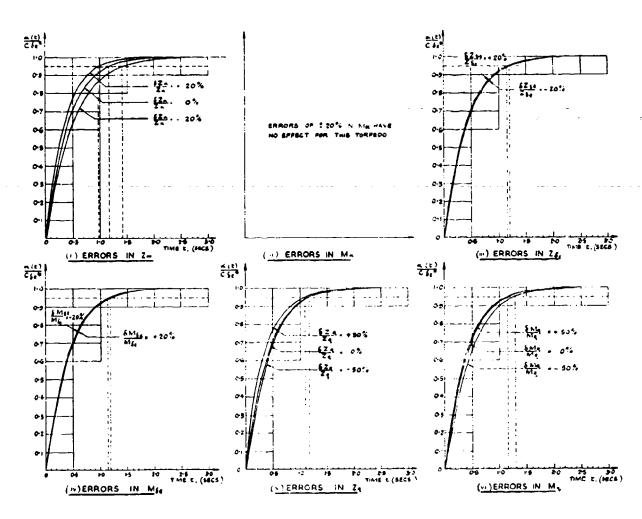


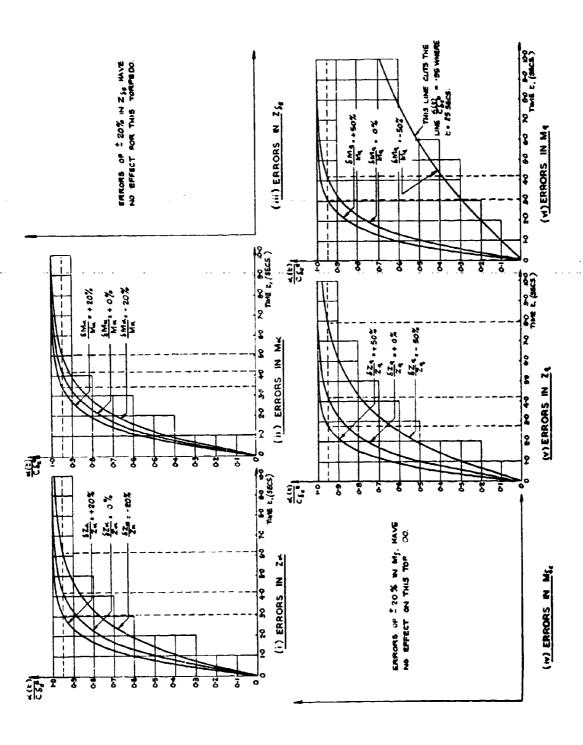
Fig. 11. Variation of decay constants  $\mu_1$  and  $\mu_2$  with  $\delta Z_x/Z_x$  and  $\delta M_g/M_g$  (Torpedo B).



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 $\kappa(\epsilon)$  ; angle of attack following step function input on elevators  $\delta_{\epsilon}^{\,\,\,\,\,\,\,\,\,\,\,\,\,\,}$  magnitude of the step function c - steady state value of angle of attack

Fig. 12. The effect of errors on torpedo motion for Totpedo A.



(t) - ANGLE OF ATTACK FOLLOWING STEP FUNCTION INPUT ON ELEVATORS
- MAGNITUDE OF THE STEP FUNCTION.
- STEADY STATE VALUE OF ANGLE OF ATTACK,

Fig. 13. The effect of errors on torpedo motion for Torpedo B.

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